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Open vehicle routing problem with moving shipments at the cross-docking terminal

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Abstract In this study, one of the internal operations at the Cross-docking Terminal (CDT), moving shipments (MS) from receiving doors to shipping doors of CDT is integrated with Open Vehicle Routing Problem (OVRP) and the problem is indicated here as OVRPCD-MS. As an additional feature, asymmetric distance between any two customers is assigned by incorporating a characteristic of one-way routes between cities in real-life transportation. The objective is to minimize the total transportation cost which incurs travelling cost between customers, service cost at customer points, service cost at the receiving and shipping doors of CDT, cost of moving shipments inside the CDT and finally the cost of hiring fleets of vehicles. To solve the OVRPCD-MS problem, a Mixed Integer Linear Programming (MILP) model is developed. The programming models are implemented in LINGO (version 18) optimization software. Branch and Bound algorithm is employed to solve ten small-scale instances generated randomly. The applicability of the proposed MILP model is observed. The required fleets of vehicles to be hired and run time to reach the optimal solution are determined. The study revealed that the average run time is exponential for small-scale instances. Thus, it can be concluded that this proposed model can be used for last time planning for small-scale instances. Also, the combinatorial nature of the vehicle routing problem makes OVRPCD-MS as NP-hard. Therefore, this study recommends that heuristic or meta-heuristic methods are more appropriate for the large-scale instances of OVRPCD-MS to reach near-optimum solutions.

Keywords: Cross-docking, Moving shipments, Open vehicle routing.

1 Introduction

The competence in performance of supply chain is essential to sustain in the competitive globalized market. To improve the efficiency of the supply chain in terms of cost and time, from time to time researchers and practitioners introduce new logistic strategies to the world. Cross-docking (CD) is one of the logistic strategies introduced in the 1930s. However, it became popular in the 1980s. Since 30% of the cost of a product is incurred due to the internal operations at the intermediate distribution centers in the supply chain, the cross-docking attempts to reduce the inventory cost and decrease the lead time from the traditional warehousing in the supply chain (Apte and Viswanathan, 2000). In other words, the cross-docking strategy eliminates more costly operations (such as storing the products and picking the orders) which are taken place at the usual distribution centers in the supply chain. Therefore, the primary operations of a Cross-docking Terminal (CDT) are unloading products from inbound vehicles, consolidating them inside the CDT and loading them into outbound vehicles. It has been estimated that roughly up to 70% of the warehousing cost can be reduced by employing the cross-docking strategy at these distribution centers (Vahdani and Zandieh 2010). Some of the reputed companies which succeeded by implementing the cross-docking strategy are Wal-Mart, Goodyear GB Ltd, Toyota, Eastman Kodak Co and Dots, LLC (Van Belle *et al.* 2012). Strategic, tactical and operational are the three levels of decisions at CDT, among which the operational level has several problems to be studied and the vehicle routing problem is one of them.

Vehicle Routing Problem (VRP) is one of the well-known optimization problems in the field of Operations Research and it plays an important role in supply chain management. The VRP is to find the optimal routes from a depot to a set of destinations each with business-specific constraints. After the study of (Dantzig and Ramser 1959) on vehicle routing problem, more interest is created among the researchers and practitioners. As a result, several variants of the vehicle routing problem were identified, and many characteristics were introduced to the literature on VRP. Open networking to the Vehicle Routing Problem (OVRP) is one of the variants of the VRP. In OVRP, the trips can be initiated from a destination and vehicles do not need to visit from the depot at the beginning. Also, the trips can be terminated at the last destination in the route and vehicles do not need to return to the depot at the end. OVRP is more suitable for an organization which does not have its own required vehicles and hence outsources the vehicles through third-party logistic companies.

A wide range of research on the vehicle routing problem and all levels at CDT is available sequentially. Though the vehicle routing happens outside the CDT and the consolidation process happens inside the CDT, they are interrelated and contribute to an efficient supply chain. Therefore, an integrated model of vehicle routing problem with cross-docking (VRPCD) is crucial for a fast and effective supply chain. The research on VRPCD was initiated in 2006 by the study of Lee *et al.* (2006). Later, VRPCD received significant attention among researchers and practitioners. Furthermore, the literature survey of Buakum and Wisittipanich (2019) on VRPCD recommends extending the research by considering the internal operations at CDT as well. Moreover, the aspect of Moving Shipments (MS) from receiving doors to shipping doors of CDT with VRPCD was mainly considered in Gnanapragasam and Daundasekera (2022) and formulated a model for Vehicle Routing Problem with Moving Shipments at the Cross-docking Center (VRPCD-MS). An optimization model for a variant of VRPCD-MS by incorporating the Time Windows characteristic was studied in Gnanapragasam and Daundasekera (2023). In this study, another variant of VRPCD-MS by considering the open network configuration is taken into account and the problem is referred to in this study as OVRPCD-MS. The objective of this study is to check the compatibility of this variant, and hence obtain the exact optimal solution to OVRPCD-MS for randomly generated small-scale instances.

1.1 Literature Review

There is a wide spectrum of studies on Vehicle Routing Problem available in the literature regarding supply chain. Also, research on cross-docking is conducted in all three levels of decisions at Cross-docking Terminal (CDT). On the one hand, these studies on vehicle routing problem as well as on CD were carried out sequentially. On the other hand, the problems on vehicle routing and cross-docking are interrelated to perform the supply chain more efficiently. Therefore, the study by Lee *et al.* (2006) initiated the research on integrated vehicle routing problem with cross-docking (VRPCD). There onwards, research on VRPCD received more attention among the researchers as well as practitioners. In this initial study, homogeneous vehicles were used in pickup and delivery processes with two different sets, one for each process. By relaxing the condition that the simultaneous arrival of inbound vehicles from the model defined in Lee *et al.* (2006) and introducing the time windows characteristic with dependency rules and consolidation decisions, Wen *et al.* (2009) generalized the VRPCD to make it more realistic.

Some of the characteristics of vehicle routing problem such as multi commodity, heterogeneous fleet of vehicles, splitting load and environmental factor were included with the model defined in Lee *et al.* (2006) by the later research work (Hasani-Goodarzi and Tavakkoli-Moghaddam 2012; Yin and Chuang 2016; Birim 2016; Gunawan *et al.* 2020). In addition to the assumptions made in Wen *et al.* (2009), splitting load, allowing customers to order from more than one supplier, maximizing total profit, customer satisfaction, many-to-many correspondence between suppliers and customers, heterogeneous fleet of vehicles, soft time windows and simultaneous arrivals of inbound vehicles to CDT were respectively taken into account in the following studies: Moghadam *et al.* (2014), Larioui *et al.* (2015), Baniamerian *et al.* 2018a, b, Nikolopoulou *et al.* (2016), and Fakhrzad and Sadri Esfahani (2014).

Several solution methods were applied in the literature of VRPCD to the model formulated in the initial study (Lee *et al.* 2006) to obtain a more optimum solution with less run time than the solution from the initial study. They are; Adaptive Memory Artificial Bee Colony, Simulated Annealing (SA), a modified Tabu Search algorithm from the initial study, hybrid method incorporating three meta-heuristics (Particle Swam Optimization, SA and Variable Neighborhood Search (VNS), SA, a matheuristic algorithm decomposed by two meta-heuristics (Adaptive Large Neighborhood Search (ALNS) and Set Partitioning Problem)) and ALNS were respectively employed in the following studies: Yin and Chuang (2016), Birim (2016), Liao *et al.* (2010), Vahdani *et al.* (2012), Yu *et al.* (2014) and Gunawan *et al.* (2020a and Gunawan *et al.* (2020b). A hybrid algorithm (Ant Colony System and SA), a constructive heuristic (Best Insert Heuristic and Memetic Algorithm), a modified VNS hybridized with Genetic Algorithm (GA), 2-phase GA, a constructive heuristic (with six local search methods) and a metaheuristic algorithm based on Large Neighborhood Search were used to reach a better optimal solution to the model developed in Wen *et al.* (2009) by the following studies respectively: Moghadam *et al.* (2014), Larioui *et al.* (2015), Baniamerian *et al.* (2018a); (Baniamerian *et al.* (2018b), Morais *et al.* (2014) and Grangier *et al.* (2017). The optimal solutions for the model constructed in Wen *et al.* (2009) to the small-scale instances were also tried out by other studies (Hasani-Goodarzi and Tavakkoli-Moghaddam (2012); Santos *et al.* (2011a); Dondo (2013); Santos *et al.* (2011b) utilizing the following solution methods: using *GAMS* software, *CPLEX* solver, 2-Branch and Price algorithms and newer Column Generation technique respectively.

The open network configuration to VRPCD (OVRPCD) was setup in the study by Yu *et al.* (2016) in addition to the model formulated in the initial study (Lee *et al.*, 2006). SA method was applied to solve OVRPCD. It was revealed that, for large-scale instances, SA outperformed the results obtained by solving the mathematical model in *CPLEX* software, in terms of both accuracy of the solution and the run time to reach the solution. The open and closed network configurations were compared in the study of multi-source VRPCD in Tarantilis (2013) using Adaptive multi-restart Tabu Search method. It concluded that, significant cost reductions were observed between closed and open network configurations. In the study of Alinaghian *et al.* (2016), open vehicle routing problem at the pickup process and closed vehicle routing problem at the delivery process were considered. The results from two different strategies incorporated in SA were compared in this study. Furthermore, the study under investigation integrates an internal operation in CDT to OVRPCD and attempts to reach exact optimal solutions to the small-scale instances of OVRPCD-MS model by applying Branch and Bound algorithm.

1.2 Problem Description

In this section, the problem under investigation is described by explaining all the processes involved in the Open Vehicle Routing with Moving Shipments at the Cross-docking Center (OVRPCD-MS) model.

Generally, in the vehicle routing problem, the depot functions as a distribution center to distribute products to the end users, or as a collection center in case of collecting products from manufacturers. However, CDT at VRPCD, functions as a trans-shipment center. In the pickup process, CDT receives the products from suppliers at the first phase, whereas, CDT delivers the products to customers at the second phase. Figure 1 represents the entire process of OVRPCD-MS.

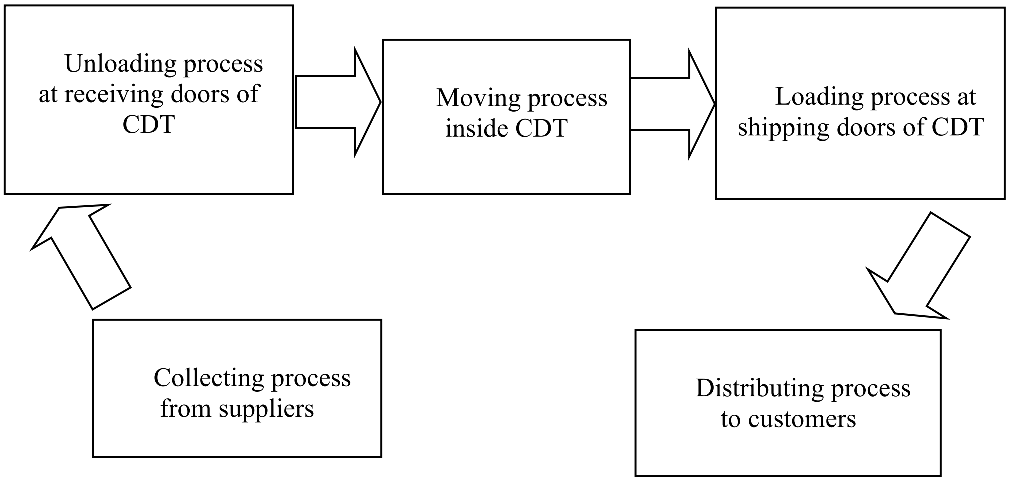

Fig 1. The process of the open network configuration with VRPCD-MS

Initially, the products are **collected** from all suppliers by inbound vehicles. Each inbound vehicle starts its route among one of the suppliers (does not need to go from the CDT at the beginning for the collecting route) and ends its route at the receiving doors of the CDT. Then at the receiving doors of CDT, accumulated products by each and every inbound vehicle are **unloaded**. Now all the unloaded products from all the inbound vehicles are sorted and **moved** to shipping doors of CDT. Consequently, consolidated products as per the requirements of customers are **loaded** into outbound vehicles at the shipping doors of CDT. Finally, the loaded products are **distributed** to relevant customers as per the scheduled route. In this case, all the outbound vehicles start their route from the CDT and end their routes after serving the last customer in that route (does not need to come back to the CDT at the end of the distributing route).

2 Methods

The formulation of the Open Vehicle Routing with Moving Shipments at the Cross-docking Center (OVRPCD-MS) model, the data used to test the model, and the method to solve the model are explained in this section.

2.1 Mixed Integer Linear Programming Model

The formulation of the Mixed Integer Linear Programming (MILP) model to the problem of OVRPCD-MS is portrayed as follows:

2.1.1 Indices

i, j : Indices for suppliers in collecting or customers in distributing process
h : Index for receiving or shipping doors of CDT
k : Index for inbound or outbound vehicles

2.1.2 Sets

$S = \{S_1, S_2, ..., S_n\}$: Set of n suppliers in the collecting process
$C = \{C_1, C_2, ..., C_{n'}\}$: Set of n' customers in the distributing process
$N = S \cup C$: Set of $(n + n')$ suppliers and customers
$V_S = \{v_1^S, v_2^S, ..., v_m^S\}$: Set of m inbound vehicles used in the collecting process
$V_C = \{v_1^C, v_2^C, ..., v_{m'}^C\}$: Set of m' outbound vehicles used in the distributing process
$V = V_S \cup V_C$: Set of $(m + m')$ inbound and outbound vehicles
$O = \{o, o'\}$: Set of receiving (o) and shipping (o') doors of CDT

2.1.3 Parameters

tc_{ij} : Transportation cost between destinations i and j
q_i : Quantity at supplier or customer
Q_S : Maximum capacity of inbound vehicles
Q_C : Maximum capacity of outbound vehicles
HC_S^k : Hiring cost of the inbound vehicle k
HC_C^k : Hiring cost of the outbound vehicle k
SC_i^k : Loading (unloading) cost at supplier (customer) i by vehicle k
SC_h^k : Unloading (loading) cost at receiving (shipping) door h by vehicle k
A_c : Fixed cost of preparation for loading/ unloading products
B_c : Variable cost of loading/unloading per unit product

2.1.4 Variables

$$x_{ij}^k = \begin{cases} 1, & \text{if vehicle } k \text{ travels from supplier (customer) } i \text{ to supplier (customer) } j \\ 0, & \text{otherwise} \end{cases}$$

2.1.5 Constraints

The constraints from (1) to (9) with their logic behind them are reported in this sub-section as follows:

All the inbound vehicles arrive at CDT from suppliers:

$$\sum_{i\in N} x_{ih}^{k} \le 1 \qquad \forall\, k \in V\ ,\ \forall h \in O \tag{1}$$

All the outbound vehicles leave from CDT to customers:

$$\sum_{j\in N} x_{hj}^{k} \le 1 \qquad \forall\, k \in V\ ,\ \forall h \in O \tag{2}$$

Only one vehicle has to arrive at one supplier or one customer:

$$\sum_{i\in N\cup O}\sum_{k\in V} x_{ij}^{k} = 1 \qquad \forall\, j \in N \tag{3}$$

Only one vehicle has to leave at one supplier or one customer:

$$\sum_{j\in N\cup O}\sum_{k\in V} x_{ij}^{k} = 1 \qquad \forall\, i \in N \tag{4}$$

Loops in routes are prevented:

$$x_{ii}^{k} = 0 \qquad \forall\, i \in N \cup O,\quad \forall k \in V \tag{5}$$

Backward movements in routes are prevented:

$$x_{ij}^{k} + x_{ji}^{k} \le 1 \quad \forall\, i,\ j \in N \cup O\ ,\ \forall k \in V \tag{6}$$

The equilibrium condition of the total supply by all the suppliers must be equal to the total demand from all the customers:

$$\sum_{i\in S} q_i = \sum_{i\in C} q_i \tag{7}$$

Maximum capacities of inbound vehicles:

$$\sum_{\substack{i\in S \\ j\in S\cup\{o\}}} q_i x_{ij}^{k} \le Q_S \qquad \forall\, k \in V_S \tag{8}$$

Maximum capacities of outbound vehicles:

$$\sum_{\substack{i\in C \\ j\in C\cup\{o'\}}} q_i x_{ij}^{k} \le Q_C \qquad \forall\, k \in V_C \tag{9}$$

The required output by satisfying the aforementioned constraints from (1) to (9) is described in the following sub-sections 2.2 and 2.3.

2.2 Calculation of required number of vehicles

The required number of inbound vehicles to collect the total supply from all the suppliers and required number of outbound vehicles to distribute the total demand by all the customers is defined in (10) and (11) below respectively:

Required number of inbound vehicles, $m = \sum_{k\in V_S}\sum_{i\in S} x_{io}^{k}$ (10)

Required number of outbound vehicles, $m' = \sum_{k \in V_C} \sum_{j \in C} x_{o'j}^k$ (11)

2.3 Calculation of the elements of the overall cost

The elements of the overall cost acquired when performing all the processes described in Section 3 are defined below from (12) to (17):

Cost of transportation at the collecting and distributing processes:

$$TC_P = \sum_{k \in V} \sum_{i,j \in N \cup O} tc_{ij} x_{ij}^k \quad (12)$$

Cost of loading (or unloading) at suppliers (or customers):

$$SC_P = \sum_{k \in V} \sum_{\substack{i \in N \cup O \\ j \in N}} SC_j^k x_{ij}^k \,, \quad \text{where}$$

$$SC_j^k = A_c + B_c \, q_j x_{ij}^k \qquad \forall i \in N \cup O, \; \forall j \in N, \; \forall k \in V \quad (13)$$

Cost of unloading at the receiving doors of CDT:

$$SC_S = \sum_{k \in V_S} \sum_{i \in N} SC_o^k x_{io}^k \,, \quad \text{where} \quad SC_o^k = A_c + B_c \sum_{\substack{i \in N \\ j \in N \cup \{o\}}} q_i x_{ij}^k \qquad \forall k \in V_S \quad (14)$$

Cost of loading at the shipping doors of CDT:

$$SC_C = \sum_{k \in V_C} \sum_{i \in N} SC_{o'}^k x_{o'i}^k \,, \quad \text{where}$$

$$SC_{o'}^k = A_c + B_c \sum_{\substack{i \in N \\ j \in N \cup \{o'\}}} q_i x_{ij}^k \qquad \forall k \in V_C \quad (15)$$

Cost of moving shipments from receiving doors to shipping doors of CDT:

$$MC_T = \sum_{k \in V_S} \sum_{\substack{i \in S \\ j \in S \cup \{o\}}} q_i x_{ij}^k \quad (16)$$

Cost of hiring vehicles:

$$HC_P = \sum_{k \in V_S} \sum_{i \in S} HC_S^k x_{io}^k + \sum_{k \in V_C} \sum_{j \in C} HC_C^k x_{o'j}^k \quad (17)$$

Therefore, the objective function of the MILP model minimizes the overall cost incurred by its elements defined above from (12) to (17). Hence, it can be expressed as follows:

Minimizing

$$Overall\ Cost = TC_P + SC_P + SC_S + SC_C + MC_T + HC_P$$

2.4 Method and Data

As defined in the previous sub-sections 2.1 to 2.3, a single objective Mixed Integer Linear Programming (MILP) model is developed to solve the OVRPCD-MS problem. Since the *LINGO* optimization software is competent of handling high-complexity optimization problems in small-scale, the above depicted MILP model is coded in *LINGO* (version 18). Moreover, as the method of solving the problem, Branch and Bound (BandB) algorithm is employed. In view of the fact that the BandB algorithm always tries to reach optimal solution and since *LINGO* is not capable of handling large-scale problems, only the small-scale instances are tried out to test the feasibility of the model.

The test instances of the small-scale are generated randomly by the following parameter estimates exhibited in Table 1. It must be emphasized that the choice of these values is highly subjective based on the practical situation and resource availability. Therefore, in this study, these values are randomly chosen mainly to generate the test instances to check the feasibility of the developed MILP for the OVRPCD-MS problem. All experiments are run on Intel Core i5 with 2.30 GHz CPU and 4 GB RAM:

Table 1. Parameter values for test instances for MILP model of OVRPCD-MS.

Parameter	Value	Parameter	Value
Transportation cost $\left(tc_{ij}\right)$	Uniform (50, 200)	Quantity $\left(q_i\right)$	Uniform (20,50)
Fixed cost for preparation to (un)load $\left(A_c\right)$	10	Variable cost for (un)loading $\left(B_c\right)$	1
Capacity of inbound vehicles $\left(Q_S\right)$	80	Capacity of outbound vehicles $\left(Q_C\right)$	50
Hiring cost of inbound vehicles $\left(HC_S^k\right)$	150	Hiring cost of outbound vehicles $\left(HC_C^k\right)$	100

3 Results and Discussion

The results obtained from the BandB algorithm implemented in *LINGO* for the tested instances are discussed in this section in detail.

3.1 Results of an instance having 4-suppliers and 6-customers

Figure 2 depicts the schedules of the routes of the optimal solution of a particular instance with 4 suppliers and 6 customers, generated based on the parameter values mentioned in Table 1. The quantity of supply by each supplier, demand by each customer and accumulated quantity in each route are clearly indicated with the sequence of suppliers/customers:

Inbound vehicle routes in the process of collecting products

$$v_1^S : S_3[35] \xrightarrow{86} S_2[37] \xrightarrow{106} CDT[72]$$

$$v_2^S : S_4[30] \xrightarrow{126} S_1[48] \xrightarrow{95} CDT[78]$$

Outbound vehicle routes in the process of distributing products

$$v_1^C : CDT[50] \xrightarrow{81} C_1[27] \xrightarrow{62} C_3[23]$$

$$v_2^C : CDT[43] \xrightarrow{50} C_5[21] \xrightarrow{78} C_6[22]$$

$$v_3^C : CDT[29] \xrightarrow{100} C_4[29]$$

$$v_4^C : CDT[28] \xrightarrow{160} C_2[28]$$

Fig. 2. The optimal solution to the instance with 4-suppliers and 6-customers

The indications in the above Figure 2 are as follows:

$\xrightarrow{tc}$: Transportation cost between two destinations $[q]$: quantity

It can be observed from Figure 2 that, 2-inbound vehicles (v_1^S *and* v_2^S) are used to collect 150 (72+78) units of products from 4 suppliers (S_1 to S_4), whereas to distribute them (50+43+29+28) to 6 customers (C_1 *to* C_6), 4 outbound vehicles (v_1^C *to* v_4^C) are utilized. The route-wise costs of all 6 elements of overall cost are represented in Table 2 in detail:

Table 2. Route-wise costs of the elements of the overall cost.

Route	TC_P	SC_P	SC_S	SC_C	MC_T	HC_P	Total Cost
v_1^S	86+106	45+47	82	-	72	150	588
v_2^S	126+95	40+58	88	-	78	150	635
v_1^C	81+62	37+33	-	60	-	100	373
v_2^C	50+78	31+32	-	53	-	100	344
v_3^C	100	39	-	39	-	100	278
v_4^C	160	38	-	38	-	100	336
			Optimal Overall Cost				2554

It can be concluded from Table 2 that the optimal overall cost of the instance with 4-suppliers and 6-customers is 2554 cost units. Further, not only the element-wise costs, but also process-wise (collecting, unloading, moving, loading and distributing process) costs can be extracted from the Table 2.

3.2 Results of small-scale test instances of OVRPCD-MS

Based on the number of suppliers (n) with their quantity of supply and customers (n') with their quantity of demand, the total quantity ($\sum_{i\in S} q_i$) supplied by all the suppliers (which is the same as total quantity ordered by customers) in the open network can be decided as the total flow of the network. To collect the total quantity ($\sum_{i\in S} q_i$) from n suppliers, the required number of inbound vehicles (m) are determined. In a similar way, the required number of outbound vehicles (m'), to deliver the total quantity ($\sum_{i\in C} q_i$) to n' customers are also calculated. Eventually, the optimal overall cost (OC_{Opt}), which contains all 6 elements described earlier in the sub-section 2.3 is computed. The average run time (T) in seconds to reach OC_{Opt} is measured. It should be emphasized that T is estimated by executing the same instance 10 times to calculate the average run time. The relevant results of all 10 test instances are summarized in Table 3.

The feasibility of the proposed MILP model can be observed from the results of the above 10 small-scale instances reported in Table 3. Since the average run time is reasonably less for the small-scale instances considered in this study, it can be concluded that this proposed MILP model can be used for last-time planning for such small-scale instances. It also can be observed that the average run time gradually increases when the number of suppliers and customers grows. Therefore, to examine the trend of increasing run time against the size of the problem, the convergence analysis is carried out and presented in the next sub-section 3.3.

Table 3. The optimal solutions to small-scale instances of OVRPCD-MS

Instance	$\sum_{i\in S} q_i$	n	m	n'	m'	OC_{Opt}	T (s)
01	150	04	02	06	04	2554	0.176
02	160	04	03	07	04	2781	0.203
03	170	05	03	07	04	3161	0.225
04	180	06	03	07	05	3412	0.279
05	190	06	03	08	04	3243	0.536
06	200	07	03	08	05	3463	1.180
07	210	08	03	08	07	3985	1.830
08	220	08	04	09	05	4108	2.679
09	230	08	04	10	05	3926	4.186
10	240	09	03	10	05	3888	6.206

3.3 Rate of Convergence

In this sub-section, how long the time will take to reach the exact optimal solution when increasing the size of the problem is analyzed. Figure 3 represents the plot of the average run time T (in seconds) against the total number $x = (n + n')$ of the suppliers and customers as the size of the problem. Since the run time for the instances of problem size less than 10 are insignificant, they are not being taken into consideration for this analysis, and therefore, in the Table 3 only 10 instances with problem size ranging from 10 to 19 are considered to plot the graph.

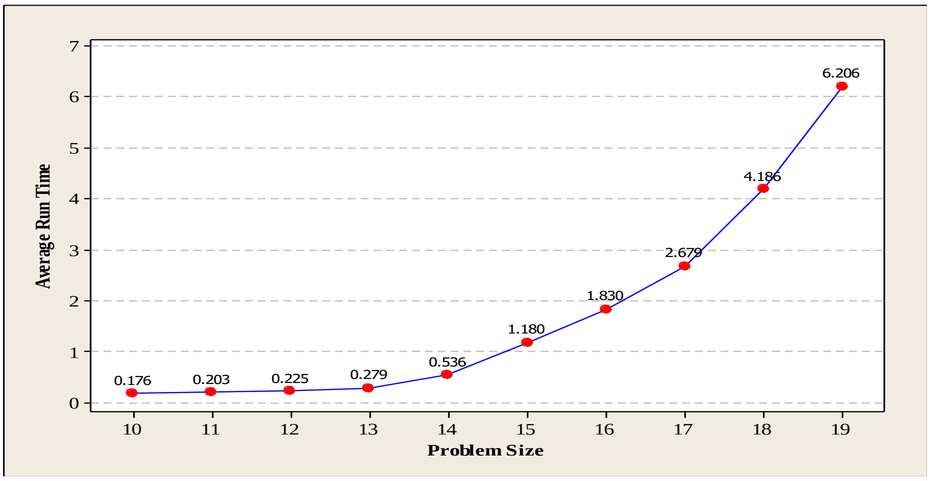

Fig. 3. Plot of average run time (*T*) versus Problem size (*x*)

It can be clearly observed from Figure 3 that the average run time to reach the exact optimal solution increases when the size of the problem increases. The fitted curve to the data points in the Fig 3 is $T(x) = 10^{-3}e^{0.436x}$ and therefore, it is evident that when the number of suppliers and customers increase, consequently the run time to reach the optimal solution increases exponentially. It reveals that the rate of convergence is $T(x) = O(e^x)$ and hence it can be concluded that, the complexity of the OVRPCD-MS problem in terms of run time is exponential. It is emphasized that, the goodness of the fitted curve is measured using the coefficient of determination value R^2 and it is approximated to 97%.

4 Conclusions

In this study, the open network configuration is adapted with the Vehicle Routing Problem with Moving Shipments at the Cross-docking Terminal (VRPCD-MS). A Mixed Integer Linear Programming (MILP) model is proposed for the Open VRPCD-MS (OVRPCD-MS). The small scale instances are generated randomly to test the applicability of the proposed mathematical MILP model. Since the mean run-time to obtain the exact solution is reasonably less for the OVRPCD-MS, it is concluded that, the developed MILP model can be applied to schedule the routes just before start of the time window. Furthermore, the rate of convergence to reach the exact solutions approaches to exponential with the increase of the instance size. Since the vehicle routing problem is a NP-hard, OVRPCD-MS is also as so. Therefore, it is recommended to apply heuristic or meta-heuristic approaches to find near optimal solutions to higher scale instances of OVRPCD-MS in a reasonably low run-time. The study further recommends that to incorporate constraints relevant to limitations on hiring of fleets of vehicles and capacity of storage at the cross-docking terminal to make the OVRPCD-MS more realistic.

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Conflict of Interest
The authors have no conflicts of interest to declare that are relevant to the content of this article

Supplementary Material
https://www.rjs.ruh.ac.lk/uploadsrjs/supfiles/5537/

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